

# Spectral spacing correlations of nucleus-nucleus collisions at high energies

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We propose a novel approach to the analysis of experimental data obtained in relativistic nucleus-nucleus collisions which borrows from methods developed within the context of Random Matrix Theory. It is applied to the detection of correlations in momentum distributions of emitted particles. We find good agreement between the results obtained in this way and a standard analysis based on the two-pair correlation function often used in high energy physics. The method introduced here is free from unwanted background contributions.

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There is currently an enormous effort underway to detect signals of possible transitions between different phases of a composite system produced in high energy nucleus-nucleus collisions (cf [1, 2]). It is anticipated that in central collisions, at energies that are and will be soon available at SPS(CERN), RHIC(BNL) and LHC(CERN), the nuclear density may exceed the density of stable nuclei by an order of magnitude. Under such extreme conditions, according to a generally held beliefs, the final product of a heavy ion collision would be a composite system that consists of free nucleons, quarks and a quark-gluon plasma. Various methods have been proposed to identify possible manifestations of such a quark-gluon plasma. Often though, results based on such methods are sensitive to assumptions made concerning the background measurements and mechanisms included in the corresponding model. In addition, the identification of the quark-gluon plasma is made more difficult due to a large multiplicity of secondary particles created at these collisions. The formulation of a reliable criterion for the selection of meaningful signals is, indeed, an important objective in relativistic heavy ion collisions physics.

In a preliminary report [3], we suggested that tools from Random Matrix Theory (RMT) [4] might be useful in illuminating the presence of correlations in the spectral (momentum) distribution of secondary particles produced in nucleus-nucleus collisions at high energy. The RMT approach does not depend on the background of measurements and relies only on fundamental symmetries preserved in heavy-ion collisions. Furthermore, the larger the multiplicity, the better the applicability of the RMT tools, and thus, its predictive power. In the present paper, we demonstrate that the RMT analysis is very sensitive to spectral spacing correlations present in the

nucleus-nucleus collision data, more so than the standard tools used for such an analysis.

Here, we make use of the experimental data that have been obtained with the 2-m propane bubble chamber of LHE, JINR [5, 6]. The chamber, placed in a magnetic field of 1.5 T, was exposed to beams of light relativistic nuclei at the Dubna Synchrophasotron. Nearly all secondary particles, emitted at a  $4\pi$  total solid angle, were detected in the chamber. All negative particles, except those identified as electrons, are considered as  $\pi^-$ -mesons. The contamination from misidentified electrons and negative strange particles does not exceed 5% and 1%, respectively. The average minimum momentum for pion registration is about 70 MeV/c. The protons were selected by a statistical method applied to all positive particles with a momentum of  $|p| > 500$  MeV/c (we identified slow protons with  $|p| \leq 700$  MeV/c by ionization in the chamber). In this experiment, there are 37792  $^{12}\text{C}^{12}\text{C}$  interaction events at a momentum of 4.2A GeV/c (for greater discussion of the details see [6]) containing 7740 events with more than ten tracks of charged particles.

The basis of our approach derives from RMT [4], which was originally introduced to explain the statistical fluctuations of neutron resonances in compound nuclei [7] (see also Ref. 8). There the precise heavy-compound-nuclear Hamiltonian is unknown or rather poorly known, and there is a large number of strongly interacting degrees of freedom. Wigner first suggested replacing it by an ensemble of Hamiltonians which describe all possible interactions [9]. The theory assumes that the Hamiltonian belongs to an ensemble of random matrices that are consistent with the fundamental symmetries of the system. In particular, since the nuclear interaction preserves time-reversal symmetry, the relevant ensemble is the Gaussian Orthogonal Ensemble (GOE). Whereas if time-reversal symmetry were broken, the Gaussian Unitary Ensemble (GUE) would be the relevant ensemble. The GOE and GUE correspond to ensembles of real symmetric matrices and of complex Hermitian matrices, respectively. Besides

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the general symmetry considerations, no other property of the system under consideration is taken into account.

If one supposes that the momenta distributions of secondary particles produced in nucleus-nucleus collisions may be treated in analogy with eigenstates of a composite system, just like the eigenstates of the compound nucleus, then the statistical analysis methods introduced by Dyson and Mehta can be applied to the LHE collision data [7]. The difference between energy and momentum is not essential for pions, and we assume that the proton mass does not significantly affect the correlation function. Note also, that here we are dealing with the momentum distribution in the target rest frame only, postponing its comparison to that in the center of mass frame, which is more natural for description of interaction. Based on this supposition, the ordered sequence of “energy levels”  $\{E_i\}$ ,  $i = 1, \dots, N$  comes from the momentum distribution and it has an average density of states denoted  $\rho_{av}(E)$ . From this sequence a new one is obtained by the unfolding procedure of the original spectrum  $\{E_i\}$  through the mapping  $E \rightarrow x$

$$x_i = \int_0^{E_i} \rho_{av}(E') dE' = \int_0^{x_i} dx' = \zeta(E_i), \quad i = 1, \dots, N \quad (1)$$

Here,  $\zeta(E)$  is the smooth function giving the mean number of eigenvalues less than or equal to  $E$  of the exact eigenvalue distribution  $N(E)$ , which is often referred to as the staircase function due to its appearance (see Ref.3). The smooth part  $\zeta(E)$  can be determined either from semiclassical arguments or by using a polynomial/spline interpolation for the exact staircase function. The momentum distribution data (see Fig.1 in Ref.3) were approximated by a polynomial function of sixth order and the distributions of various spacings  $s_i$  from the 7740 events satisfy the condition of  $\chi^2$  per degree of freedom is less than unity.

The effect of the mapping is that the sequence  $\{x_i\}$  has on average a constant unit mean spacing (a constant unit density), irrespective of the particular form of the function  $\zeta(E)$  [11]. The spacings are defined as  $s_i = x_{i+1} - x_i$  between two adjacent points and they are collected in a histogram, which gives the probability density. If the “events”  $\{x_i\}$  are independent, then the form of the histogram must follow  $p(s) = \exp(-s)$  known as the Poisson density. On the other hand, if the levels are repelled (anticorrelated) as in the GOE, the density is approximately given by the Wigner surmise form  $p(s) = \frac{\pi}{2}s \exp(-\frac{\pi}{4}s^2)$ . Interestingly, the spacing probability density approximately follows the Wigner surmise for high energies, whereas at relatively low energies the corresponding spacing density is maximum at the origin and nearly the Poisson density [3]. In an eigenspectrum, the Poisson density arises where there is a dominance of many crossings between different eigenenergies, whereas the Wigner surmise reflects the tendency to avoid clustering of levels. In turn, the crossings are usually observed where there is no mixing between states that are char-

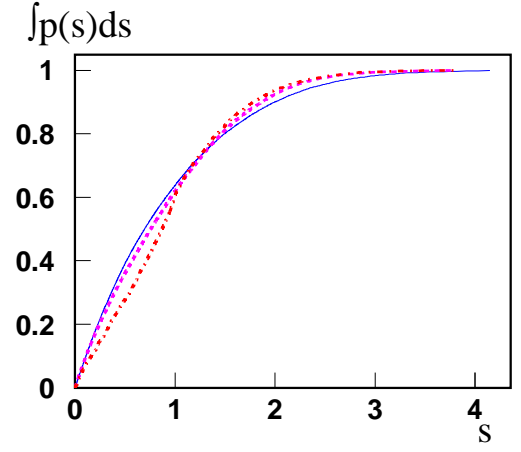


FIG. 1: (Color online) Cumulative spacing momentum distributions for different regions of measured momenta: a)  $0.1 < |p| < 1.14$  GeV/c (solid line); b)  $1.14 < |p| < 4.0$  GeV/c (dashed line); c)  $4.0 < |p| < 7.5$  GeV/c (dot-dashed line). The solid line exhibits the Poisson distribution, while the dot-dashed line is close to the Wigner-Dyson type distribution.

acterized by different good quantum numbers, and the anticrossings signal a strong mixing due to a perturbation brought about by either external or internal sources (cf Refs.7, 8).

The transition from one probability density to the other has been used, for example, in nuclear structure physics to study the stabilization of octupole deformed shapes and transition from the chaotic to the regular pattern in the classical limit [10]. Therefore, such an analysis can provide the first hint of some structural changes at different parameters of the system under consideration, in particular, in different energy (momentum) intervals. Figure 1 shows the integrated momentum spacing density for experimental data, demonstrating a gradual transition from a Poisson-like density toward a Wigner-like density with the increase of the absolute value of the momentum distributions.

In order to elucidate the degree of correlations for a stationary spectrum with unit average spacing Dyson introduced the *k-level correlation functions*

$$R_k(x_1, \dots, x_k) = \frac{N!}{(N-k)!} \int \dots \int P_k(x_1, \dots, x_N) dx_{k+1} \dots dx_N \quad 1 \leq k \leq N, \quad (2)$$

where  $P_k(x_1, \dots, x_N) dx_1 \dots dx_N$  gives the probability of having one eigenvalue at  $x_1$ , another at  $x_2, \dots$ , another at  $x_N$  each within the interval  $\{x_i, x_i + dx_i\}$ . By integrating  $P_k(x_1, \dots, x_N)$  over all variables but one, in the limit  $N \rightarrow \infty$ , one obtains the ensemble averaged density

$$\tilde{\rho}(x_1) = \int \dots \int P_k(x_1, \dots, x_N) dx_2 \dots dx_N \quad (3)$$

which is normalised to unity. From Eq.(2) it follows that  $R_1(x_1) = N\tilde{\rho}(x_1)$  and  $R_k(x_1, \dots, x_k) dx_1 \dots dx_k$  is the probability, irrespective of the index, of finding one level

within of each of the intervals  $[x_i, x_i + ds]$ . From the above definition it follows that  $R_1(x) = 1$ . With the aid of the definition (2), by integrating  $R_{k+1}$  one obtains

$$\int R_{k+1}(x_1, \dots, x_{k+1}) dx_{k+1} = (N - k) R_k(x_1, \dots, x_k) \quad (4)$$

It is difficult to work directly with the  $R_k$  functions. One

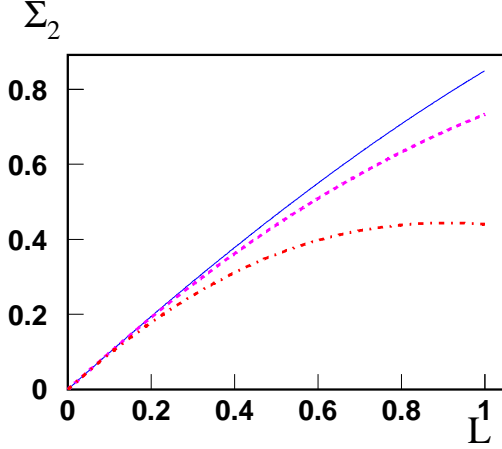


FIG. 2: (Color online) Number variance  $\Sigma^2(L)$  for three different region of the experimental spacing momentum distributions: a)  $0.1 < |p| < 1.14$  GeV/c (solid line); b)  $1.14 < |p| < 4.0$  GeV/c (dashed line); c)  $4.0 < |p| < 7.5$  GeV/c (dot-dashed line).

important and more convenient measure of correlation that was introduced is based on the number statistic  $n(L)$  which is defined to be the number of levels in an energy interval of length  $L$ . Since the spectrum was unfolded, the average number statistic  $\langle n(L) \rangle = L$  is independent of the spectrum. However, the variance of  $n(L)$

$$\Sigma^2(L) = \langle [n(L) - \langle n(L) \rangle]^2 \rangle \quad (5)$$

does depend on the spectrum considered. For the Poisson density (see [4])

$$\Sigma^2(L) = L \quad , \quad (6)$$

and for the GOE, the exact asymptotic expression is

$$\Sigma^2(L) = \frac{2}{\pi^2} \left[ \ln(2\pi L) + \gamma + 1 + \frac{1}{2} [Si(\pi L)]^2 - \frac{\pi}{2} Si(\pi L) - \cos(2\pi L) - Ci(2\pi L) + \pi^2 L \left[ 1 - \frac{2}{\pi} Si(2\pi L) \right] \right] \quad (7)$$

Here  $\gamma$  is the Euler constant and  $Si$ ,  $Ci$  are the sine and cosine integrals, respectively. The number variance  $\Sigma^2(L)$  calculated using the optimal implementation of the definition in Eq.(5) is shown in Fig. 2. Indeed, this quantity manifests the Poisson statistics (Eq. (6)) for experimental spectra with a low momenta distribution. On the other hand, one again observes a clear indication on the presence of correlations for large momenta.

For the analysis of fluctuations, it is more convenient to use pure k-point functions [12]

$$\hat{R}_k(L) = \int_0^L \dots \int_0^L R_k(x_1, \dots, x_k) dx_1 \dots dx_k \quad (8)$$

The function  $\hat{R}_k(L)/k!$  gives the probability that an interval of length  $L$  (for small  $L$ ) contains  $k$  levels. In RMT most emphasis has been put on the two-point correlation function  $\hat{R}_2(x_1, x_2)$  or density-density correlation function. The two-point correlation function is the probability density to find two eigenvalues  $x_i$  and  $x_j$  at two given energies, irrespective of the position of all other eigenvalues. The function  $R_2(x_1, x_2)$  depends only on the relative variables  $s = x_1 - x_2$ . The variance  $\Sigma^2$  is connected to  $\hat{R}_2$  through the following relation

$$\begin{aligned} \hat{R}_2(L) &= \int_0^L \int_0^L R_2(x_1, x_2) dx_1 dx_2 - 2 \int_0^L (L - s) R_2(s) ds \\ &= \Sigma^2(L) + L(L - 1) \end{aligned} \quad (9)$$

The two-level correlation function  $R_2(x_1, x_2)$  determines the basic fluctuation measures related to Wigner's level repulsion and the Dyson-Mehta *long-range order*, i.e., large correlations between distant levels. Bohigas *et al* (1985) [12] provided a thorough analysis of level repulsion and long-range correlations (rigidity) for different correlation functions. To understand the distinct role played by level repulsion and long-range order in the momentum density, we compare our numerical results with analytical expressions from Table 1 of Ref.12 for the Poisson ensembles (there is neither level repulsion nor long-range order) and for the GOE case (this ensemble exhibits both features). The two-point correlation function  $\hat{R}_2(x_1, x_2)$  calculated from Eq.(9) is shown in Fig.3. Even though

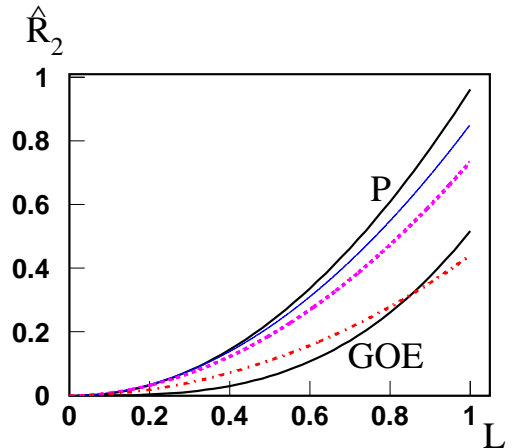


FIG. 3: (Color online) Two-point correlation function  $\hat{R}_2(s = L)$  for different regions of measured momenta: a)  $0.1 < |p| < 1.14$  GeV/c (solid line); b)  $1.14 < |p| < 4.0$  GeV/c (dashed line); c)  $4.0 < |p| < 7.5$  GeV/c (dot-dashed line). The solid lines, denoted as P and GOE, display the characteristic limits for Poisson and GOE ensembles, respectively.

there are small deviations from Poisson ( $\hat{R}_2(L) = L^2$ ) and GOE ( $\hat{R}_2(L) = \pi^2 L^3/18$ ) predictions, the experimental results for the momentum distributions reproduce surprisingly well both limits.

The validity of the RMT analysis is confirmed by an independent analysis of the data with the aid of the standard pair-correlation function (see, for example, Ref.13 and references therein):

$$R(y_1, y_2) = \sigma \frac{d^2 \sigma / dy_1 dy_2}{(d\sigma / dy_1)(d\sigma / dy_2)} - 1 \quad (10)$$

Here, the quantity  $\sigma$  is the cross section of the inclusive reaction and  $y = \frac{1}{2} \ln \frac{E+P_{||}}{E-P_{||}}$  is the rapidity, which depends on the particle energy  $E$  and its longitudinal momentum  $P_{||}$ . The rapidity is one of the main characteristics widely used in relativistic nuclear physics (see [14, 15]). The pair-correlation function manifests the difference be-

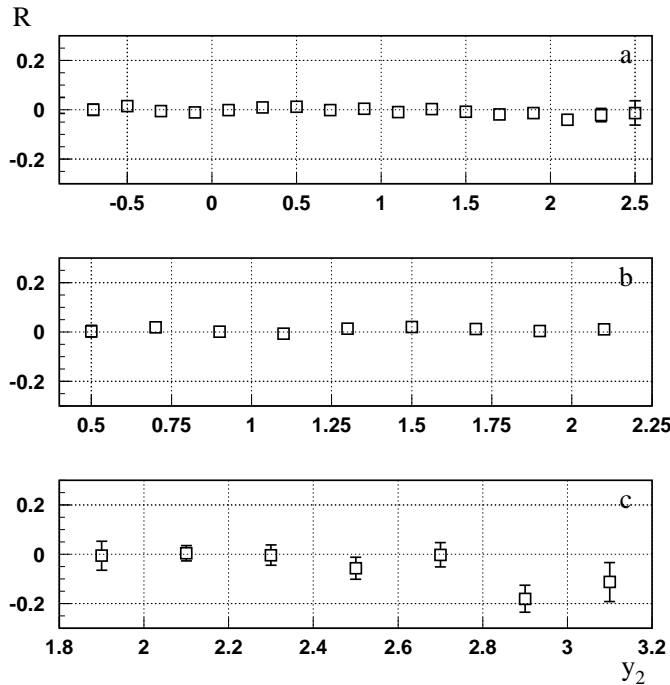


FIG. 4: Integrated two-pair correlation functions for particles obtained in CC-interactions (see the text) for different regions of measured momenta: a)  $0.1 < |p| < 1.14$  GeV/c; b)  $1.14 < |p| < 4.0$  GeV/c; c)  $4.0 < |p| < 7.5$  GeV/c.

tween probability density of two-particle events and the

product of the probability densities of independent particle events. It vanishes if the particle rapidities are independent. Figure 4 demonstrates the results for particles obtained in CC-interactions. For the sake of illustration, we integrate the function  $R(y_1, y_2)$  over one of the variables, say  $y_1$ , and consider the dependence on  $y_2$ . For different momentum distributions, there are three intervals of integration for the variable  $y_1$ : a) for  $0.1 < |p| < 1.14$  GeV/c the function  $R(y_1, y_2)$  is integrated in the interval  $-0.9 < y_1 < 2.5$ ; b) for  $1.14 < |p| < 4.0$  GeV/c it is integrated in the interval  $0.5 < y_1 < 2.4$ ; and c) for  $4.0 < |p| < 7.5$  GeV/c it is integrated in the interval  $2.5 < y_1 < 3.5$ . There are, of course, some experimental errors which are not seen in the behaviour of the function  $R = \int_{y_1} R(y_1, y_2) dy_1$  on Figs.4a,b due to the large data set. These errors do not spoil, however, the results in the region c), where experimental data on the multiparticle production exhibits an indication of the existence of correlations between particles in the region  $4.0 < |p| < 7.5$  GeV/c. The physical origin of the correlations is beyond the scope of the present paper and will be discussed elsewhere. One observes that the predictions based on the standard pair-correlation function  $R$  are consistent with the predictions based on the RMT analysis. However, the RMT two-point correlation function magnifies the presence of correlations manifested in the standard pair-correlation function (compare Figs.3 and 4c).

Summarizing, we propose an analysis of relativistic nuclear collision data based on ideas from RMT. The approach is free from various assumptions concerning the background of the measurements and it provides reliable information about correlations induced by external or internal perturbations. All these features make our proposal a quite promising avenue for the future analyses of data produced in heavy ion collision experiments.

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